

## PREFACE

It was in the year 1996, my first text book, Engineering Mathematics-III for the third semester B.E course of Mysore and Kuvempu universities was published. Since then the book has undergone three revisions as per the VTU syllabus which is revised once in four years. I am completing **fifteen years of career in the field of text book writing** without looking back. My esteemed readers on one side and the trusted publishers on the otherside are responsible for this memorable milestone. I am ever grateful to them in making this possible.

The current book **ENGINEERING MATHEMATICS - III** prepared with all care is in accordance with the latest VTU syllabus. It also caters to the need of autonomous institutions in Karnataka and other technological universities in the country.

I hope the book will live upto the expectations of the community of my beloved readers.

My sincere thanks to two of my senior colleagues in the department, **Dr. D. Mamta** and **Smt. G.V. Pankaja** for their scrupulous scrutiny of the content in the book.

I heartily thank **Mr. K.V. Balakrishna** of **M/s. Sudha Publications** for all the encouragement, publishing my books for all these years.

I appreciate the highly professional computer type setting work by **Sri. S. Raghunandhan** and team of 'Allkind' and thank them.

I also thank the printers for the quality printing.

I profusely thank the readers community, specially the students for the appreciative nice words through telephone and SMS.

I humbly invite comments and suggestions from all corners for further improvement of the book.

**August 1st 2011**  
(Shravana Paadya)  
**Mysore - 8**

**K.S.Chandrashekar**

## REWARD

*VTU students of the current scheme w.e.f 2010-11, scoring 125/125 in all four papers of Engineering Mathematics I to IV Semesters (10 MAT 11, 21, 31, 41) will be rewarded with a cash prize of Rs.7,500/- by the author. Please write to the author directly along with attested xerox copies of marks cards of all the four semesters.*

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**Achiever** : **Mr. Bharath M.V**, a student from 2006 batch of E&C branch from PESIT, Bangalore, received cash prize during 2008.

## SYLLABUS

### ENGINEERING MATHEMATICS - III

Code : 10 MAT 31  
Hrs / week : 04  
Total Hrs : 52

IA Marks : 25  
Exam Hrs : 03  
Exam Marks : 100

#### PART - A

##### **Unit - I : FOURIER SERIES**

Convergence and divergence of infinite series of positive terms, definition and illustrative examples, Periodic functions, Dirichlet's conditions, Fourier series of periodic functions of period  $2\pi$  and arbitrary period, half range Fourier series. Complex form of Fourier Series. Practical harmonic analysis. [7 hours]

##### **Unit - II : FOURIER TRANSFORMS**

Infinite Fourier transform, Fourier sine and cosine transforms, properties, Inverse transforms [6 hours]

##### **Unit - III : APPLICATIONS OF P.D.E**

Various possible solutions of one dimensional wave and heat equations, two dimensional Laplace's equation by the method of separation of variables, Solution of all these equations with specified boundary conditions. D'Alembert's solution of one dimensional wave equation. [6 hours]

##### **Unit - IV : CURVE FITTING AND OPTIMIZATION**

Curve fitting by the method of least squares - Fitting of curves of the form  $y = ax + b$ ,  $y = ax^2 + bx + c$ ,  $y = ae^{bx}$ ,  $y = ax^b$

**Optimization** : Linear programming, mathematical formulation of linear programming problem (LPP), Graphical method and simplex method. [7 hours]

## PART - B

### **UNIT - V : NUMERICAL METHODS - 1**

Numerical solution of algebraic and transcendental equations : Regula falsi method, Newton - Raphson method, Iterative methods of solution of a system of equations: Gauss - Seidel and Relaxation methods. Largest eigen value and the corresponding eigen vector by Rayleigh's power method. **[6 hours]**

### **UNIT - VI : NUMERICAL METHODS - 2**

Finite differences : Forward and backward differences, Newton's forward and backward interpolation formulae. Divided differences - Newton's divided difference formula, Lagrange's interpolation formula and inverse interpolation formula.

Numerical integration : Simpson's one-third, three-eighth and Weddle's rules (All formulae / rules without proof) **[7 hours]**

### **UNIT - VII : NUMERICAL METHODS - 3**

Numerical solutions of PDE - finite difference approximation to derivatives, Numerical solution of two dimensional Laplace's equation, one dimensional heat and wave equations. **[7 hours]**

### **UNIT - VIII : DIFFERENCE EQUATIONS AND Z-TRANSFORMS**

Difference equations : Basic definition; Z-transforms - definition, standard Z-transforms, damping rule, shifting rule, initial value and final value theorems. Inverse Z-transform. Application of Z-transforms to solve difference equations. **[6 hours]**

**Note : In the case of illustrative examples, questions are not to be set.**

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